# Abstract

Bayesian analysis in inverse modeling aims to compute the probability distribution of uncertain parameters, constrained by forward (PDE) models, given a prior knowledge on the unknown parameter, and noisy observations. Prominent examples include Bayesian inverse problems in medicine, biology, imaging, and subsurface flow as well as climate and weather forecasts. One of the main questions arises here is that how much and how optimal information one can extract through a Bayesian approach from given measurement data in an experiment. In this dissertation, we answer this question by developing optimal experimental design (OED) methodologies for Bayesian inverse problems. To this end, the expected information gain (EIG) is calculated as an optimality criterion which specifically is the expected logarithmic ratio between the posterior and prior distributions. Optimizing this ratio leads to optimal designs of experiments with the most informative data. This results in the reduction of uncertainty in the Bayesian parameter of interest in the mathematical model.

The expected information gain includes nested integrals and usually does not have a closed form to compute. In this dissertation, double loop Monte Carlo sampling methods are developed to estimate the EIG in real world applications such as electrical impedance tomography (EIT) inverse problem in medical imaging, where the goal is to find the optimal electrode configuration. For this inverse problem, a nonlinear elliptic partial differential equation, namely, Poisson-Boltzmann equation is used as the corresponding forward PDE model. Moreover, as double-loop sampling methods are inherently computationally expensive, a Laplace approximation technique is developed to overcome this issue and to compute the EIG in a faster and more efficient way. Furthermore, a variance reduction approach namely the multilevel quasi Monte Carlo method for the efficient approximation of the EIG is developed.

Bayesian inversion and optimal experimental design have various applications in computational science and engineering, medicine and biology. In this dissertation, as another application, the Bayesian inverse modeling of epidemiological diseases is explored. In order to model the spread of infectious diseases, a SIR (susceptible-infected-removed) system of ordinary differential equations is developed and Bayesian inversion techniques are deployed for the robust and reliable estimation of the unknown parameters in the epidemiological model.