Abstract

Empirical integral transform methods are powerful yet relatively little-known tools of statistical inference. They offer a framework for designing estimators and tests in parametric and non-parametric settings. The procedures typically rely on a distance measure between the transform of a model and its empirical counterpart, or employ empirical versions of some unique transform properties that hold under null hypotheses.

This work advances parametric estimation of probability distributions using their Laplace transforms and characteristic functions. Specifically, differential equations satisfied by the transforms are used to construct estimators that are both robust and explicit, while retaining comparatively high efficiency - a rare feature among diverse types of existing estimators. A method is presented for deriving the equations, enabling applications to distributions with intractable transform expressions.

The main analytical effort lies in establishing the asymptotic normality and robustness theory of the proposed estimators, with robustness examined through influence functions. Expressions for asymptotic covariance matrices and influence functions often involve intricate integrals, which appears to be the cost for achieving explicitness in the estimators themselves.

The thesis places equal emphasis on empirical evidence. Extensive simulations are conducted to compare the proposed estimators with popular robust and non-robust techniques. Various distribution types are considered, including symmetric and skewed ones, with light and heavy tails, further in the presence of outliers and model misspecifications. The combination of experiments and theoretical analysis reveals a crucial finding: an optimal trade-off between efficiency and robustness of the estimators, along with their numerical reliability, can be consistently achieved by pre-estimating the scale of the estimators' weight function from the sample. The author contends that this aspect has been overlooked in early constructions of transform-based and other minimum distance estimators relying on weighted integrated distances.

The transform methods, especially based on the differential-equations, moreover enable inference for a variety of non-standard distributions. Prominent instances include mixed, compound, and non-normalized distributions encountered across diverse application fields. In addition, families of distributions, such as the Pearson or Katz family, are often characterized by differential equations, either in the variable's or transform domains. Therefore, the presented methods support estimation and identification within

entire families. While non-standard models and the families are not our primary focus, numerous examples are given to demonstrate a wider spectrum of applications and motivate further research.

A separate chapter is dedicated to goodness-of-fit testing, introducing a novel test for the log-normal distribution based on the Laplace transform. In particular, the procedure utilizes a functional differential equation satisfied by the transform. The test compares well in terms of power with several famous tests (e.g., Shapiro-Wilk, Jarque-Bera, Anderson-Darling). Importantly, it tends to be uniformly powerful across distributional alternatives and remains quite stable concerning its tuning parameter's value. Aspects such as consistency, asymptotic distribution of the test statistic, and bootstrap-based determination of critical points, are addressed.