Abstract

In this cumulative thesis, we examine the analytic properties and spherical harmonic decomposition of generating functions of rotationally equivariant Minkowski valuations.

First, we show that generating functions, which were merely known to be integrable, are in fact continuous up to the poles and differentiable almost everywhere. For weakly monotone Minkowski valuations, we describe the local behavior around the poles and conclude that the space of convex bodies with C^2 support functions gets mapped into itself. Regarding the corresponding spherical convolution transform, we extend the known spectral gap inequalities to a larger class of Minkowski valuations. As an application, we prove that for mean section operators and for even, monotone Minkowski valuations, Euclidean balls are the only fixed points in some C^2 neighborhood of the unit ball.

Second, we consider the action of Alesker's Lefschetz integration operator on the generating function. We show that the convolution kernel of the arising transform is a strictly positive function that is smooth up to the north pole. From this, we deduce that all known examples of rotationally equivariant Minkowski valuations are preserved by the Lefschetz operators.

More generally, we describe the action of the Lefschetz operators on the Klain–Schneider function of scalar valued valuations by a Radon type transform between flag manifolds, generalizing a result of Schuster and Wannerer. In the course of this, we introduce a new way to express the mixed area measure of a lower dimensional body in terms of its surface area measure relative to a subspace.

Third, we show an analogue of the Klain–Schneider theorem for valuations that are invariant under rotations around a fixed axis, called zonal, and thus, also Minkowski valuations. From this, we obtain new proofs of the representation theorems of smooth and continuous Minkowski valuations that are considerably shorter and more accessible. We also establish a new integral representation for zonal valuations, where the role of the area measure is taken by the mixed area measure with a disk, and we introduce an easy way to move between these two representations.

As applications, we give a simpler proof of the integral representation of mean section bodies by Goodey and Weil in terms of Berg's functions. In the case of origin symmetric bodies, we establish a new representation in terms of a mixed volume involving disks. Moreover, we recover and extend various integral geometric formulas.