Abstract

Design optimisation deals with the minimisation (maximisation) of a given shape functional, which maps subsets of $\mathbf{R}^d$ to the real numbers, with respect to the design variable. Therein, one distinguishes between the terminology *shape optimisation*, which is concerned with smooth deformations of a given shape and *topology optimisation*, which addresses topological changes. Put into the context of mechanical engineering, we refer to both topics as *structural optimisation*. Recent advances in the manufacturing process gave rise to a large variety of design possibilities. Consequently, the demand for optimal designs and appropriate optimisation methodologies has increased significantly.

We first follow the approach of classical shape optimisation. We apply the well established theory revolving around the shape derivative to a model problem in the framework of linear elasticity with pointwise stress constraints. These constraints are then compactly formulated by the maximum norm, which results in a nonsmooth optimisation problem. We employ methods from nonsmooth analysis to derive optimality conditions and draw a connection to the Clarke subgradient. Additionally, we consider three simple geometries to address the numerical applicability of our methodology.

Next, we investigate topological sensitivities in the framework of topology optimisation. We compare three different adjoint based methods to derive the first and second order topological derivative. We apply these methods to a PDE constrained problem in the framework of linear elasticity and highlight the differences in view of applicability and efficiency.

Based on our observations, we then employ the averaged adjoint method to compute the complete topological asymptotic expansion for a PDE constrained model problem including the Laplacian and a perturbation of the right hand side. We observe that, depending on the objective functional, the asymptotic analysis of the adjoint variable can be more involved. In fact, a L_2 tracking-type cost functional requires the introduction of the fundamental solution of the biharmonic equation.

Finally, we utilise the notion of topological state derivatives to investigate numerical schemes in the context of topology optimisation. We approximate a state of the art level-set algorithm and introduce a steepest descend scheme in the context of one-shot type methods.